Chapter 1

Basics

1.1 gaussian quadrature

(http://de.wikipedia.org/wiki/Gau%C3%9F-Quadratur)
Gaussian quadrature
$g(x)$ is the function which needs to be integrated
$g(x)$ is splitted to $g(x) = w(x) \cdot \Phi(x)$ $w(x)$ are the weights for $\Phi(x)$. $\Phi(x)$ is then approximatedby a polynom $p(x)$ with specific initial values x_i. The resulting formula can be integrated very easily:

$$\int_a^b g(x)dx = \int_a^b w(x) \cdot \Phi(x)dx \approx \int_a^b w_i \cdot p(x)dx = \sum_{i=0}^{n} \Phi(x_i)w_i$$

where $w(x) \geq 0$
$\Phi(x)$ is a continous function
the integrated guassian quadrature matches exactly for all polinomials with a $degree <= 2n - 1$

1.2 maxwell-boltzmann distribution

Chapter 2

Dimensionality

We have the following dimensional parameters of the real world:

Domain size S' is the flow domain size along the x axis. The size of one fluid cell in each dimension is then $\Delta x = S'/res$ where res specifies the grid resolution. Thus we get the following dimensional parameters:

viscosity	ν'	$\left[\frac{m^2}{s}\right]$
domain size	S'	$[m]$
grid resolution	res'	$[]$
gravitational force	g'	$\left[\frac{m}{s^2}\right]$

To limit the compressiblity the dimensional timestep is determined by

$$\Delta t' = \sqrt{\frac{g_c \cdot \Delta x'}{|g'|}}$$

A value of $g_c = 0.005$ keeps the compressibility below half percent.

The other parameters have to be adopted to a cell size of 1 size unit length and also to a time step of 1 time unit length.
Therefore we scale the viscosity and gravitational parameter:

$$\nu = \nu' \cdot \frac{\Delta t'}{\Delta x'^2}$$

$$g = g' \cdot \frac{\Delta t'^2}{\Delta x'}$$

As a result we can use a unit length for the cell size ($\Delta x = 1$) as well the timestep ($\Delta t = 1$) for the whole simulation.
The relaxation time τ and the can be computated from the viscosity:

$$\tau = 3\nu + \frac{1}{2}$$

$$\omega = \frac{1}{\tau}$$

Chapter 3

Notes

mass tracking:
keeps track of an additional variable m for the mass of the fluid in the cell
change of mass is given by fluid stream. adds of subtracts the change of fluid exchange
distingishes:

1. empty cell: $m <= 0$

2. interface cell: $0 < \mathrm{m} < \rho$

3. fluid cell: $\rho <= m$

standard exchange:

1. at least one cell is not a interface cell at x and $x + \vec{e_i}$:

$$\Delta m_i(x, t+1) = f_{\tilde{i}}(x + \vec{e_i}, t) - f_i(x, t)$$

2. both cells x and $x + \vec{e_i}$ are interface cells:

$$\Delta m_i(x, t+1) = (f_{\tilde{i}}(x + \vec{e_i}, t) - f_i(x, t)) \frac{(\epsilon(\vec{x} + \vec{e_i}, t) + \epsilon(\vec{x}, t))}{2}$$

the fluid fraction within the cell for interface cells can be calculated by

$$\frac{\epsilon = m}{\rho}$$

mass differences are summed up to represent the new mass:

$$m(x, t+1) = m(x, t+1) + \sum_{i=1}^{18} \Delta m_i(x, t+1)$$

there is no mass exchange with empty cells!

fluid density distribution which comes from empty cell has to be manually reconstructed since there cant be an exchange with empty cells (ρ_g is the density of the gas, often set to 1)

$$f_{\tilde{i}}(x, t+1) = f_i^{eq}(\rho_g, \vec{u}) + f_{\tilde{i}}^{eq}(\rho_g, \vec{u}) - f_i(x, t)$$

new fluid density coming from empty cell
    = eq. dens. distrib. **outgoing** with **fluid velocity in interface cell**
    + eq. dens. distrib. of **neighbored empty cell incoming with fluid velocity of interface cell**
    - outgoing fluid of fluid cell to empty cell

QUESTION: why does it makes sense to use the density of the air because it's thousand times less than that of air?!?

if a cell has **less than 0 or more than** ρ **mass** the remaining mass has to be redistributed to the neighbored interface cells!